Heideggers epistemic Ἀπορία and the Possibilities of its Dissolution by the Thinking of Alain Badiou

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Summary

- 1. Small introductory Remarks on the Relationship Heidegger / Badiou
- 2. Dilthey and Heidegger: The Riddle of Being and the Problem of *Weltanschauung*
- 3. Badiou and ZFC: Clarity Regained
  - 3.1 Basic Axioms of Set theory
  - 3.2 Existence Axioms
  - 3.3 Constructibility V=L and Forcing
  - 3.4 Multiverse Problem
  - 3.5 Further Problems (if there is time)
- 4. Overview / Basic Points. Three « main positions » in the foundation of science.

1.1: The Problem of Heidegger, Relativism and Irrationalism

- Today Heidegger has become a *persona non grata* in all philosophy for his National Socialist Convictions (specially in Germany)
- Nontheless his thought has become widespread, there are plenty of *open* or *secret* Heideggerians.
  - Open Heideggerians: Morton, Malabou, Nancy, Marion, Markhal, Harman, Dugin etc.
  - Secret Heideggerians: Latour, Bennett, « New Materialism », Deleuze (?)
  - Related but not identical is also *Philosophy of Life* (Vitalism), *Existencialism*, *Postmodernism*
- Commonplaces: irrationalism, relativism, positions against science, mathematics, technology, etc.
- Heideggerianism is popular, as a *general ideology,* for fascists and left-liberals alike.
- It is dangerous for ecological, political and anticapitalist ideas which all rely on *science*.

1.2: Three Key Remarks of Badiou on the Relationship Heidegger - Badiou

- 1st Manifest for Philosophy
- In « Being and Event 1 » Two Orientations: Heideggers « Thinking of Proximity » vs. Platos « Substraction »
- Badiou claims: Only « lateral » relevance and mentioning of mathematics for Heidegger

1.3: Three Main Claims

- 1. There *is* in fact a « secret » but fundamental relationship of Heidegger with Mathematics and even Set theory. They are *already* on the same floor, without knowing it.
- 2. Being and Event 1 and ZFC is a proper *answer* and *alternative* to Heideggers « Thinking of Being (Denken des Seins)»
- 3. Being and Event 3 / Immanence of Truths is a *solution* of the Aporia of Heidegger.
  - 3.1 Argument with forcing against V=L / Idealism
  - 3.2 Ontological Maximalism
- So Badious thinking can be also used against he open and secret Heideggerians. It can also be used for a foundation of *science*.

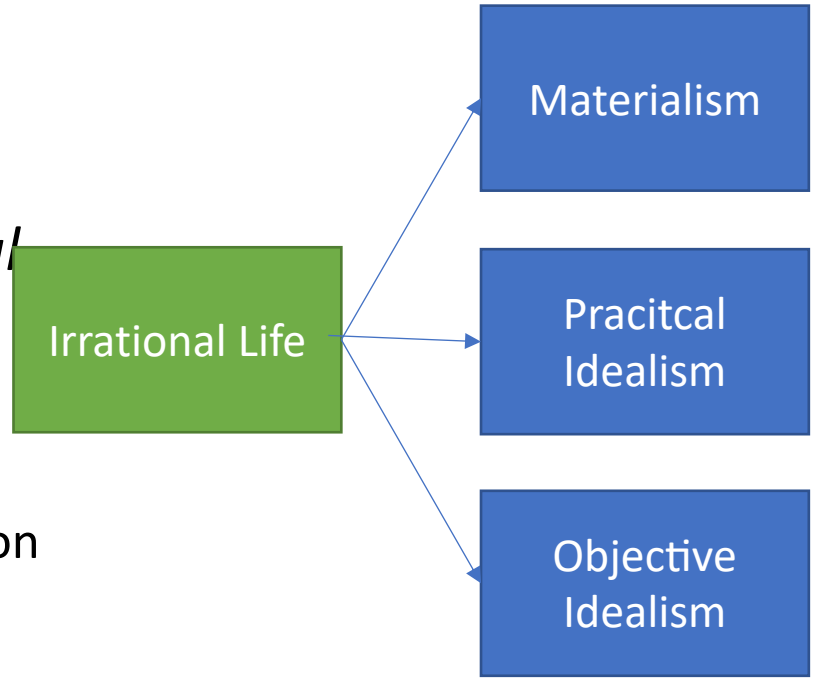
Martin Heidegger and the Thinking of Being

2.1 « Dissolution » of traditional Philosophy in the 19. Century

- Traditionally, philosophy is supposed to *grasp* and *know* highest principles which are sufficient to build up science. -> Plato, Leibniz, Kant, Hegel; also Hobbes, Locke etc.
  - 1. *Intuition* of highest principles
  - 2. or *experience* of highest principles
  - 3. Self-Foundation
- For *Dilthey* this adventure clearly has come to an end. *Hegel* is for him the last big philosopher wo attempted this, but his system *failed*.
- For Dilthey, everything emerges out of the irrational *life*. *Life-Philosophy, Vitalism*.

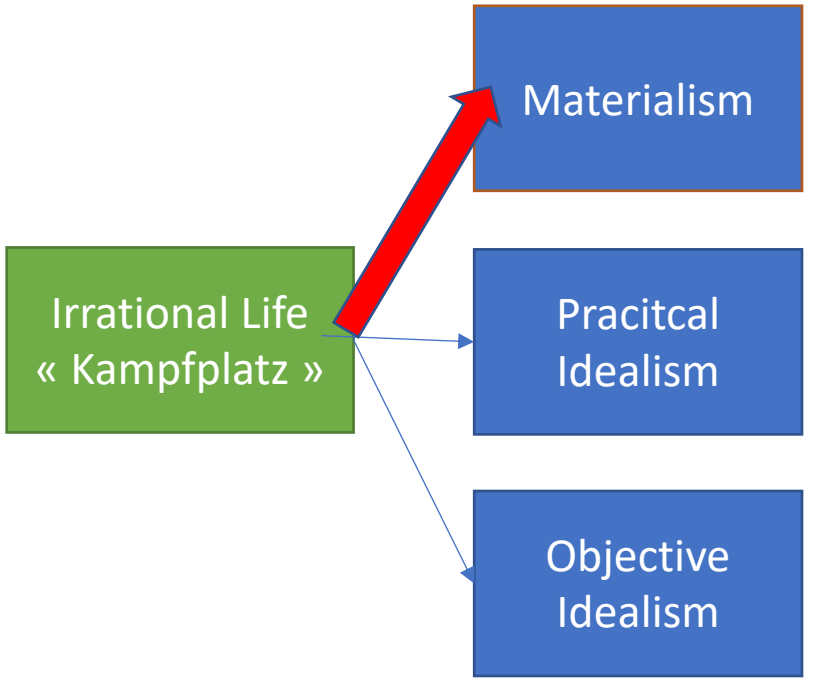
2.2 Diltheys »Weltanschauungslehre »

- The *Summit* of this relativist ideas was Diltheys *Weltanschauungslehre*. In it, Philosophy, Religion and Art emerge *irrationally* out *life*.
- There is no possibility to *decide on a rational basis* between
  - 1. Materialism (Hobbes, Epikur)
  - 2. Practical Idealism (Kant, Plato)
  - 3. Objective Idealism (Hegel, Spinoza)
  - (which all can not found themselves or build upon evidences)
- What decides, is the irrational, animalic Life ifself. It expresses itself in « Weltanschauung »



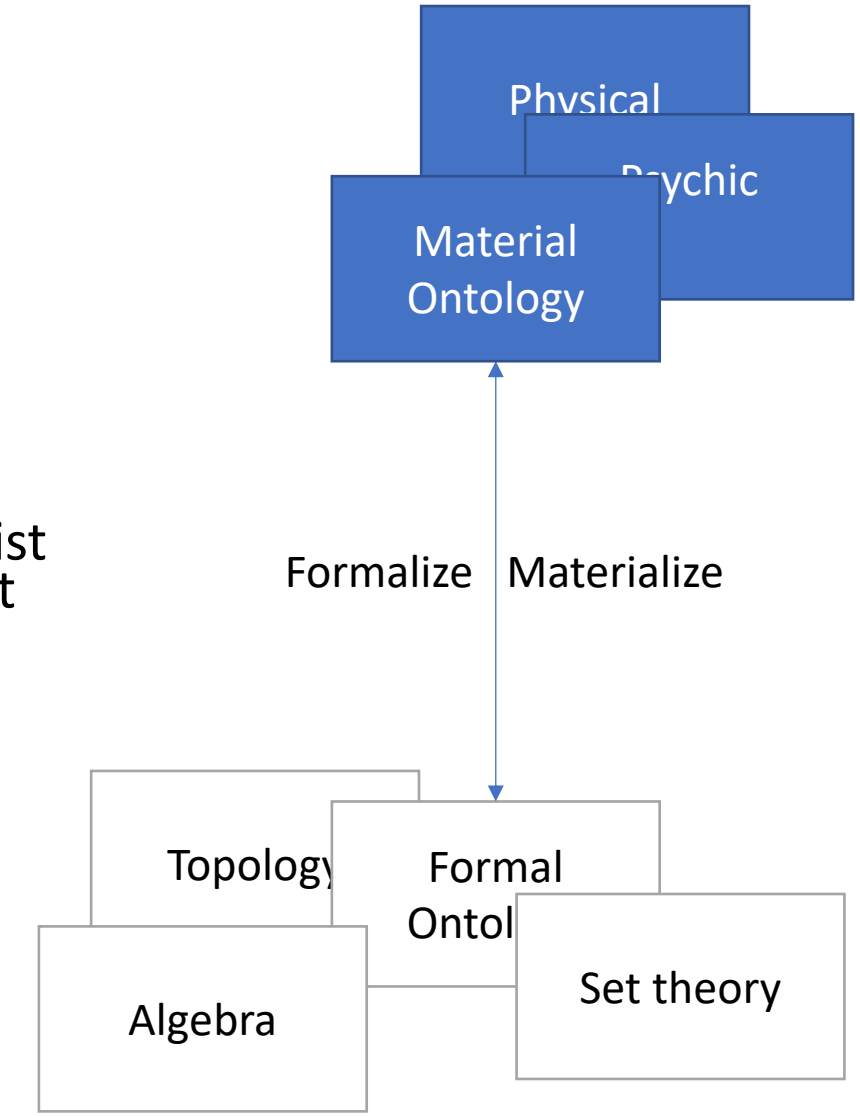
2.3 Similar Ideas in Marxism, but another *Attitude*

- *Engels* famousely claimed, that Philosophy is not moving by its own thought, but passively reacts to the discoveries of *science*.
- *Lenin* used this in *Materialism and Empirocriticism.* Philosophy is kind of battleground that it has to *decide* for the correct, materialist side. Only in Materialism we have « full » science.
- *Althusser* developed this further, with *Philosophy and the spontaneus Philosophy of Scientists.*
- General Idea:
  - 1. Philosophy is not a science, it needs a *new Praxis*.
  - 2. Chosing between Idealism and materialism is a decision without proper *reasons*. Its a *fight* of almost political character.
  - 3. We nonetheless must *take* this decision in favor of materialism / for science, for opening up a discourse which is not *arbitary* anymore. Idealism would « open up » things that are « outside » of science.



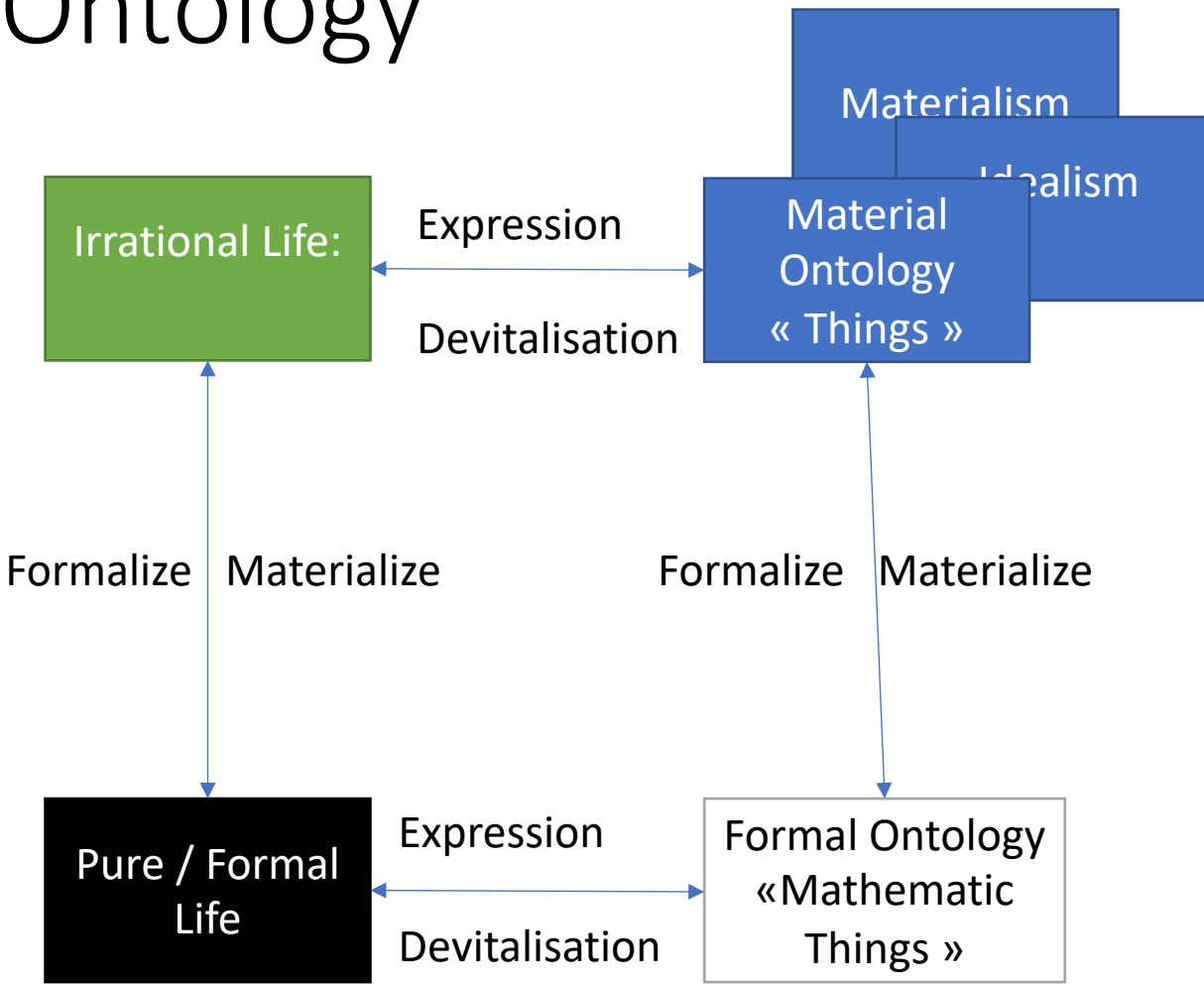
2.5 Husserls Division of formal and material Ontologies

- Another important Source of Heidegger was Husserls Division between *formal* and *material* Ontologies.
- Material Ontologies are for example
  - 1. Psychological Objects
  - 2. Physical Objects
  - 3. Spiritual (Geistige) Objects
- Formal Ontologies, on the other side, are arithmetical mathematical systems, show *laws* and *properties* that exist for all material ontologies and then specially *set theory*. It expresses, what « Being itself » or « Something as such » is. Specially also *Set theory* is named here.
- The operation for the »extraction» of the formal-ontological object is called « formalisation » and the contrary « materialization »



2.6 Heideggers Fourfold Ontology

- In the *first* Lecture after the 1 World War, Heidegger already the *Core* of his later thought.
- Its an *Extension* of the vitalist Dualism of Dilthey……with the Dualism of formal and material Ontology of Husserl. So we get *4 Positions!*
- In *Being an Time*, this pure Life is later called *Autenticity « Eigentlichkeit »*. It is a *formalized* Life, the experience of *Being* itself (Sein).
- Important is the new Form: Pure / Formal Life. It will be called *autenticity*. It consists of different *Aspects*.
  - 1. *Anxiety* (Angst, Angoisse) shows pure Life / Time / Dasein, *before* it has any direction
  - 2. *Being towards Death* (Vorlauf in den Tod), which is the anticipation of the impossibility of anticipate something, and *emerges* out of anxiety.
  - 3. *Resolutness* (Entschlossenheit).



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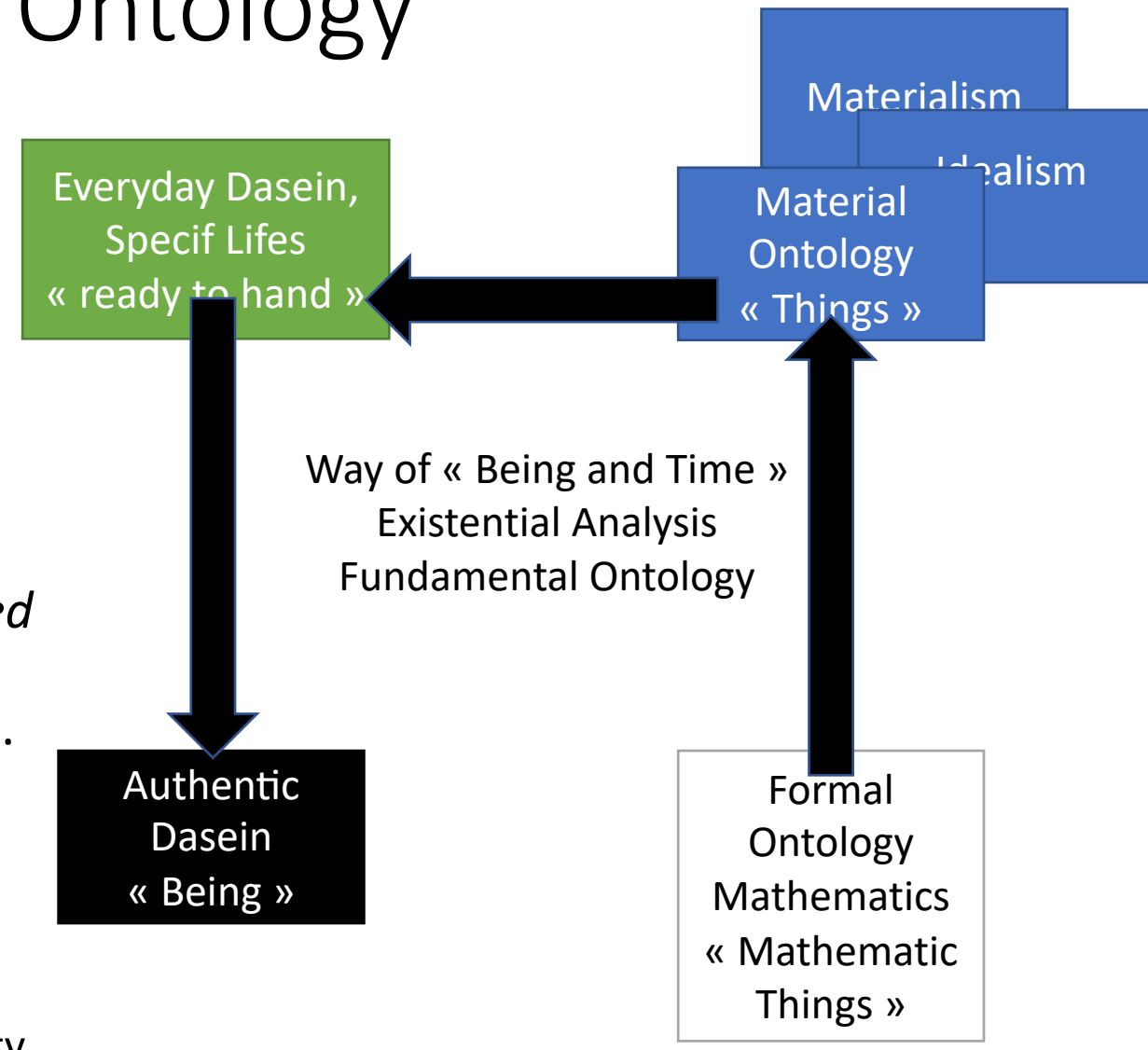

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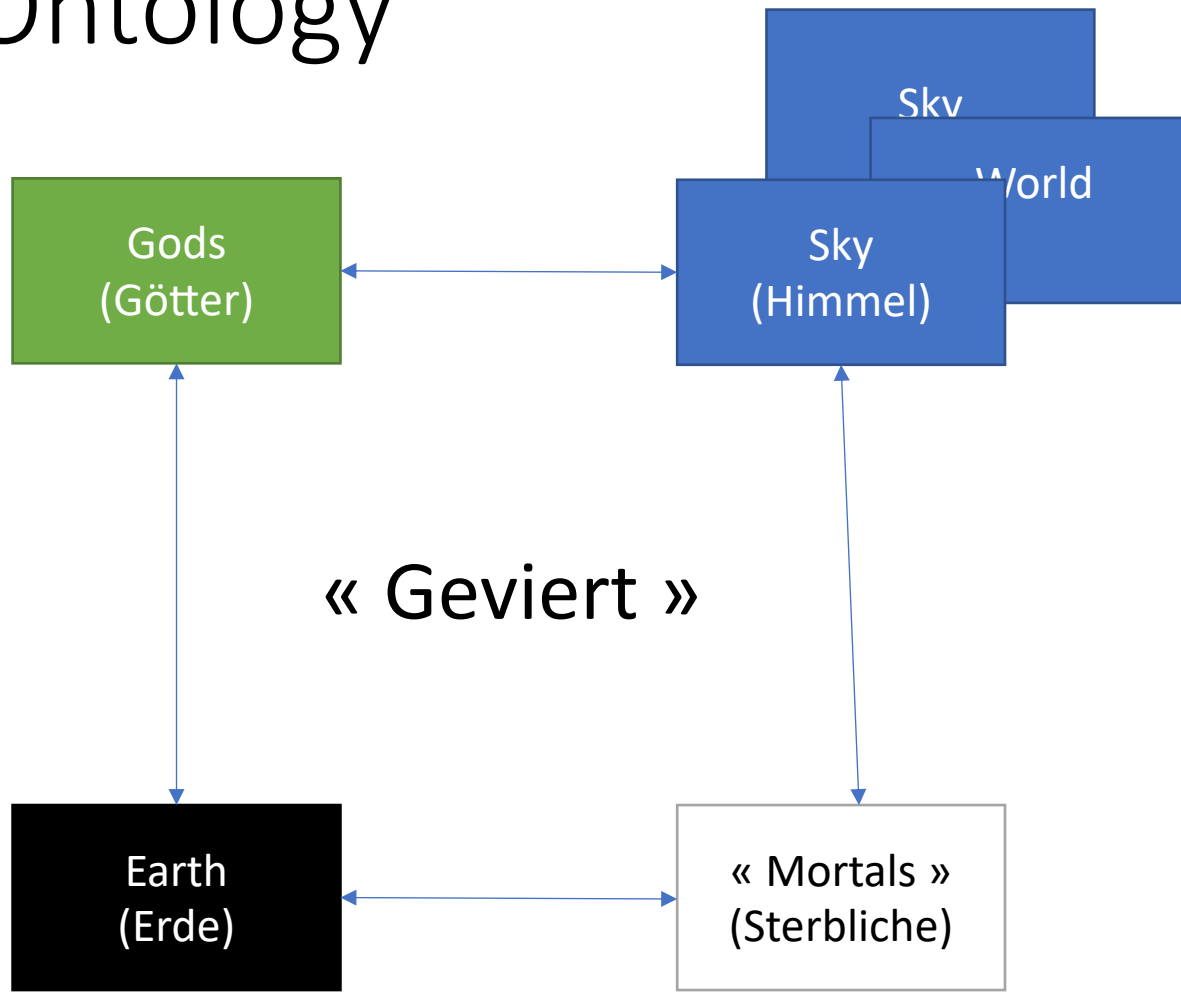


2.7 Being towards Death (Vorlauf in den Tod)

- « Going towards Death » is in *Being and Time* of crucial importance. Its the *authentic « true » understanding,*
- which consist in the *acceptance* or *anticipation of*
  - 1. Your own death
  - 2. Your death at any possible moment
  - 3. Therefore: The impossibility of understanding
  - 4. The impossibility of understanding *at any possible Moment*
- Furthermore (in the backround)
  - 1. The end of the world (Apocalypsis)
  - 2. The end of the world *at any possible moment* (Apocalypsis, Hora Incerta)
  - 3. The Return or not-Return of God (Openess for Religion and Atheism).
- *It means: Not* having any specific « Weltanschauung »
  - 1. Neither Materialist, nor Idealist
  - 2. Also neither Religious or Not-Religious
  - 3. Just the *Experience of Openness, pure Contingency:* « Thinking of Being / Denken des Seins »

2.8 *Resoluteness (Entschlossenheit)*

- This « absolute Openness » of Being towards Death ist inherently unstable. Its *impossible* to « stay » in that openness of the « pure Riddle ».
- In « Being and Time » a crucial aspect *third aspect* of *Authenticity* is « Resoluteness (Entschlossenheit)», which consists in an act of *consciousness* (Gewissen), to *decide*. This enables *in Authenticity* to relate to a *specific Being;* from « Being (Sein) » to « specific Beings (Seiendes) in the *Moment of vision* (Augenblick).
- In later works this is thought in the Notion of *Truths* (Wahrheit), of the *Event* (Ereignis) and ultimately the *Founding* (Gründung) of a new world.
- But its clear: this *Decision for a specific Being* out of an *Event* for the *Founding* of a new world has no *reasons* to decide in the one or other direction. Its *arbitary or irrational*. « We are all resolute, but we don't know for what ».
- We see, this *decision for a specific being or world* is nothing else than the decision for a worldview (Weltanschauung), that is for example, Materialism, Idealism, Christianity etc.

2.9 *Science and Truth* for Heidegger

- This *relativist* conviction of switching to a specific different *»Weltanschauung»* affects deeply also his notion of *science*.
- In his later text on the work of art, his position towards science and philosophy is put clear:
  - 1. *Science* (Wissenschaft) isn´t the attempt to decide and to reach *truth*, but the search for « *correctness ».* Because of this, Science *can not think*. Its a *dependent* and *blind* activity.
  - *2. Philosophy*, specially *metaphysics or ontology* is the moment of decision. Its the founding (Gründung) of a world, in which science can happen, and it necessary has to decide on Being, but *without knowing it,* so they think they are using « intuitive truths »
  - 3. The *Thinking of Being* (Denken des Seins) on the other thinks the undecided Being and also that its an act of decison – its a kind of « *Meta-Philosophy »* which installs itself « above » the specific philosophies.
  - 4. And will end up in a specific science and specific « Weltanschaunng. »
- It can be seen that Heideggers « Thinking of Being » is at the same time
  - 1. Defining a kind of pluralism or relativism because there are many « Weltanschauungen »
  - 2. At the same time « totalitarian », because the Weltanschauung
  - 3. Deeply Irrationalist.

Alain Badiou and the « Dissolution » of the Riddle of Being

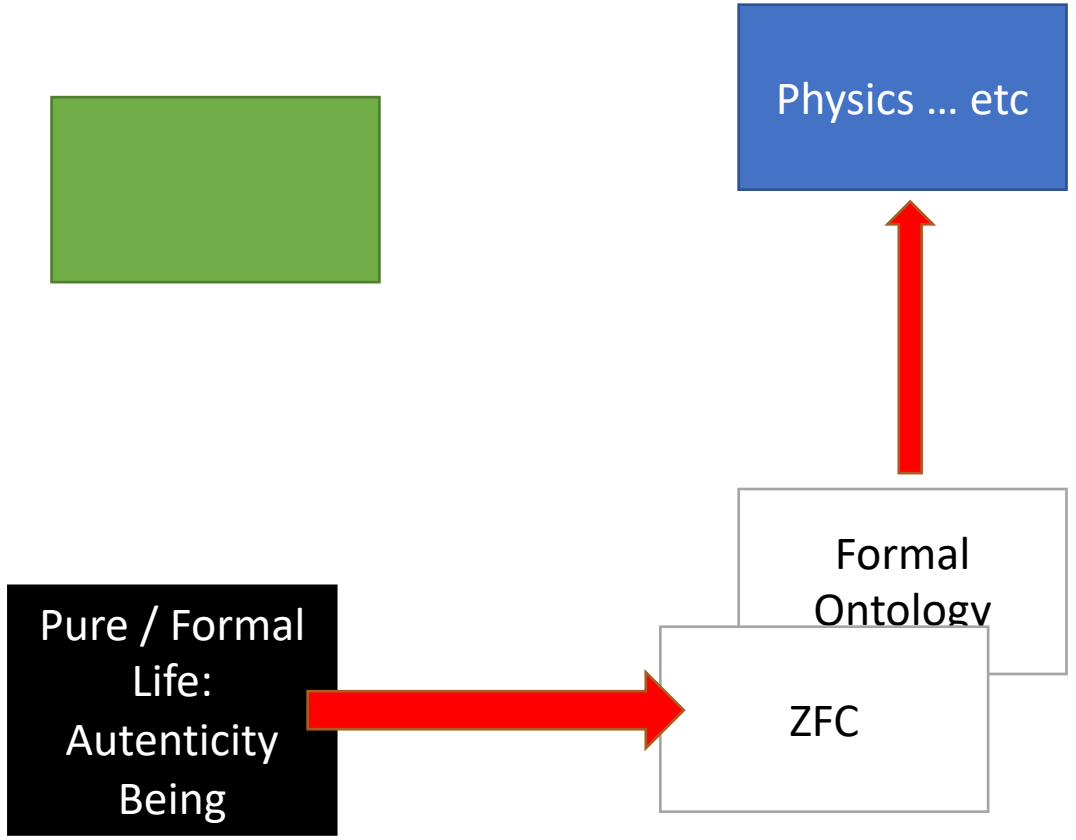
3.0.1 Introductory Remarks

- The Ontology of Badiou in *Being and Event 1 and 3* is build upon *ZFC* and its extensions, but draws also a lot out of *Heidegger* and *Althusser*.
- It is *opposed* to Heidegger in a specific sense, but completely aware of the problems that he describes. It is *on the same side of Althusser, Lenin, Engels,* but tries to find *better* arguments in ZFC.
- For that, there is a certain « translation » of philosophical notions of Heidegger and others to problems of Set theory and ZFC. Translation means:
  - 1. Mathematical notions have a philosophial meaning.
  - 2. Philosophical meaning has a mathematical notons.
- Generally, in *Being and Event 1* the *discussion* of the use of one axiom is the very center: **V=L,** and then the **Forcing procedure and generic sets** as main argument *against* it.
- Both problems can give *a lot* of new philosophical impulses and perspectives to the « problem » of Heidegger; that is the arbitarity of the ontological decisions. And it's *not* that easy: In a lot of aspects forcing can also *strengthen* Heideggers position, in the so-called *multiverse* theory.

1. Being and Event has a notion of pure Being (Sein, Etre), which either is *not* mathematizable, in the undecidedness of questions or in the notion of *V* (True world of sets).
2. Being and Event has a notion of *stratified beings (Seiendes, Etant),* that is, *sets* a, b, c. But they have a *priority against Being (Sein, Etre).* In Heideggers terminology they would be « formal beings ».
3. It builds upon *decisions*, that is: *axioms of ZFC* and its possible *extensions by new axioms* that can be known by forcing and other specific methods.

3.0.2 Main Points, Overview

- 1. General idea of axiomatisation in Set theory and mathematics in general
- 2. Existence axioms: Empty Set, Infinity etc.
- 3. Importance of forcing for the axiomatisation of set theory and its meaning for « decisions » in ontology.
- 4. The problem of the « mathematical multiverse » and « ontological maximalism »
- (Optional: 5. Open questions: Axiom of Choice, Axiom of Foundation and general determinacy)



3.1.1 Set-Centrism? ZFC-Centrism?

- The basic meta-axiom that Badiou poses is the well known: ***Mathematics = Ontology*.**
- Badiou talks in his works indeed a lot about mathematics in general, but in Being and Event 1 only about **set theory**, and then only **ZFC** is considered. But Its clear, mathematics is not = set theory, and set theory not ZFC. Why this focus?
- The *relevance* of set theory in the practice of mathematics can not be underestimated, but nonetheless – its just a human tool that is specially often used.
  - 1. The usefulness of set theory is, that it is very easy to use it as a *foundation,* that is, the « world of sets » is place where « everything » can be modeled. Instead of speaking of numbers, algebras or geometric objects, we can just talk about *specific sets*
  - 2. Also Sets are very « poorly structured ». Most mathematical objects, like for example topological spaces, graphs, algebra are « sets with extra structure ». It looks more « primitive » or « elementary »
  - 3. Cardinality and therefore infinity can be studied there « more easily »
  - 4. It has *not* found the « true » world of sets, it has no standard model.
- Now ZFC is just *one* set theory. But also ZFC has certain useful properties. Question »Why ZFC and not another theory» is answered in the discussion of its axioms. The question, why these axioms and not others, is of great relevance. Here Badiou offers a *new heideggerian solution*.
- Generally, set theory and then ZFC was developed as a 1. theory of foundation, 2. to the research of infinity and for the decision of the *continuum hypothesis,* which is a *main question* regarding the nature of infinity

3.1.1 Axiomatisation of geometry and set theory by *intuition* and their *limits*.

- Generally, we see that not only in philosophy, but also in mathematics *guiding intuitions* are starting to lack to build theories, they are switching more and more to *decisions*. This is especially true of set theory.
- First example of an axiomatization of intuition: *Euclidic Geometry* (300 BC?)
  - There is an sensible « intuition » to guide the axiomatisation.
  - Kant's *transcentental Aesthetics* and *transcendental Judgment faculty* speak about this. Kants Philosophy is *generally* trying to build science upon « pure intuition », and here on the *intuition of space*.
- Riemann Geometry (1854) and others, Topology
  - There are Geometries that are *not* intuitive, are consistent objects of science.
  - Our « Euclid » world is just *an accident! There are way more geometries!*
  - So for a « general » geometry intuition can not be *only* source of axioms, its getting *more symbolic*.
- Hilberts (1899) and Tarskis (1926) Geometry
  - Attempt to erase all relation to intuition and build up everything on formal Axiomatic Systems (FAS).
- Set Theory and axiomatisation after Cantor and others (End of 19. century)
  - Now *sets* are even more abstract than topological spaces. There seems to be *no direct intuition* to guide the axiomatization of set theory, because sets are *not immediately intuitive*.
  - Gödel still defended the « intuition » of set theory axioms or of their Gödelian « intuition » as guiding principle of axiomatic decisions in mathematics, and we will see, that he has good reasons for this point.

3.1.2 Arithmetics and Axiomatisation by Intuition. The Incompletess theorems.

- For Kant, Arithmetics works with another intuition, which is *Time* and our ability to *count*. So the arithmetic axioms are *also* synthetic judgements apriori, which can be the basis of *theorems*.
- Also *arithmetics* got axiomatized in the modern sense, the first attempt in the modern sense are the *Peano Axioms (PA)* 1898. The « most primitive » one is Q, Robinson Arithmetics (1950)
- The « intended model » of arithmetic theories is N, the *natural numbers*.
- Gödels Theorems 1931 now proved:
  - 1. Arithmetic systems can formulate sentences or questions that can neither be proved or disproved. Arithmetic theories have *many models* and are not *categorical*. These Sentences can be *added* to build another theory. They have « always » accidental matter and escape thought.
  - 2. The *consistency* of the system is a sentence of that kind, a system *can not* show their own consistency.
- Set theories like ZFC have exacly the *same* problem.
- And its clear, these independent statements that can be *new axioms* for another theory most often are not intuitive. We can not use a *intuition* to guide our decisions on them. Also here the « save heaven » of intuition is collapsing and the space of decisions is opening up, and then the big question, *why* should one take one decision and not another.

3.1.3 « Basic Axioms » of ZFC and their « Intuition » ?

- Where do the axioms of *set theory* come from? Do we have intuition here? Or something else? We will see, that here the problem *radicalizes*. Lets start with the « basic axioms ».
- Axioms like 1. Extensionality Axiom, 2. Union Set Axiom, 3. Pairing Axiom seem to be *intuitive statements* what can and could be set. They seem to be insofar intuitive, as they concern finite sets that can be *abstracted* from our perception, by the annihilation *structure*.
- The same goes for the 4. Power Set Axiom
- The Axiom Schemata of 5. Separation and 6. Replacement are more difficult, but could also be defended as being *indirecly intuitive*.
- Generally, we get to a similar problem like with geometries: Our *intuition* might give us a *hint* how to build a theory of sets, but ultimately it might be not enough to think all sets, and its possible that a *generalisation* is possible by giving up intuition. And even the decision to follow intuition or not is a decision. As Badiou says, ontology is starting to be based on decisions, and then the question emerges, why them and not others.
- We will see, that the problems will grow even bigger if we look at the « less basic » axioms, which have even more the character of a « decision ».

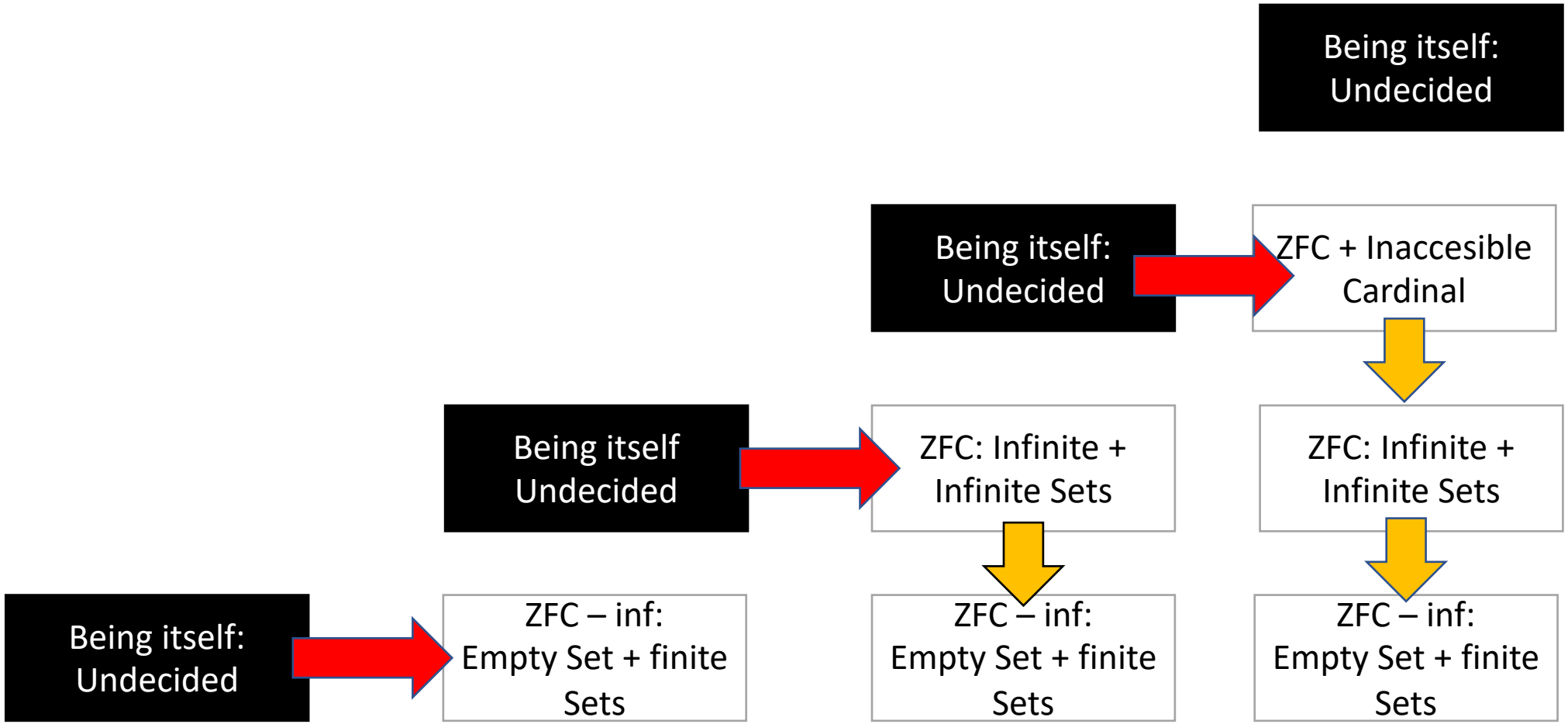
3.2 Existence Axioms

- Until now the so-called « basic axioms » allow us to think that there has to be a set if another one exists. But In the « world of sets » there is actually no single set, if no existence axiom is put.
- Two existence axioms are in the classical ZFC:
  - 1. The Axiom of the empty set states the existence of a set which has no elements
    - A. This + basic axioms is enough to build up basic finitary mathematics: Arithmetics, Algebra etc.
  - 2. The Axiom of Infinity states the Existence of an *infinite* set or the set of natural numbers.

    A. This is specially important for the foundation of real numbers, analysis, transfinite arithmetics and also the problems of the continuum hypothesis.

    B. If you don't have the Axiom of Infinity, you get **ZFC – inf**, which is the *finitist* ZFC.
- What *decision* is behind this? What Intuition or non-intuition is working here? Is it arbitrary? Clearly, the *Axiom of infinity* is completely *un-intuitive.* Rejecting it leads to *Finitism* or a finitist set theory. So its a proper *decision*, and we have to ask *what* could motivate it.
- The problem of *existence axioms* is deeply connected to the *second incompleteness theorem* we talked about.
  - 1. Remember: No System can proof its own consistency.
  - 2. But ZFC *with* infinity an proof the consistency of ZFC – inf, *without* infinity!
  - 3. Also, the statement of the *existence of infinity* is nothing else than the statement of the actuality of the just potentially infinite finite sets in the finitist system.
  - 4. So the existence axioms form a guiding principle in building up the set-theoretic universe.

3.2 Existence Axioms



3.3.1 Foundation, Choice, Constructivity V=L

- We switch to the less intuitive Axioms of classic ZFC. The discussion of the addition 8. Axiom of Foundation and 9. The Axiom of Choice is at the same time the defence of a *non-intuitionist* set theory, in which intuitionism is avoided and *classical* mathematics is possible.
- But now its also relevant to ask, why it shouldn't be better to use another, even stronger axiom, the *axiom of constructivity* V=L? This would be plausible, because the Axiom of Foundation and the Axiom of Choice *follow* from V=L. This is called the *constructivist orientation* in Being and Event 1.
- Classical ZFC-Set theory *does not* use V=L, and has this axioms, and is therefore *in between* of two extremes…
  - 1. Not using V=L, and also not Choice and Foundation as its consequences (Intuitionism).
  - 2. Using Axiom of constructivity V=L, and therefore using Foundation and Choice as consequences or Theorems (Constructivism).
- This *decision* against V=L is the most broadly discussed axiomatic desision in *Being and Event 1.* To understand this, we have to look closer what V=L actually means
- It clear, the *discussion* of the use of axiom of foundation and choice is also necessary, but generally this topic will pose more deep problems, which could bring Badiou also to a certain *limit* or a new *challenge* (in Part 5).

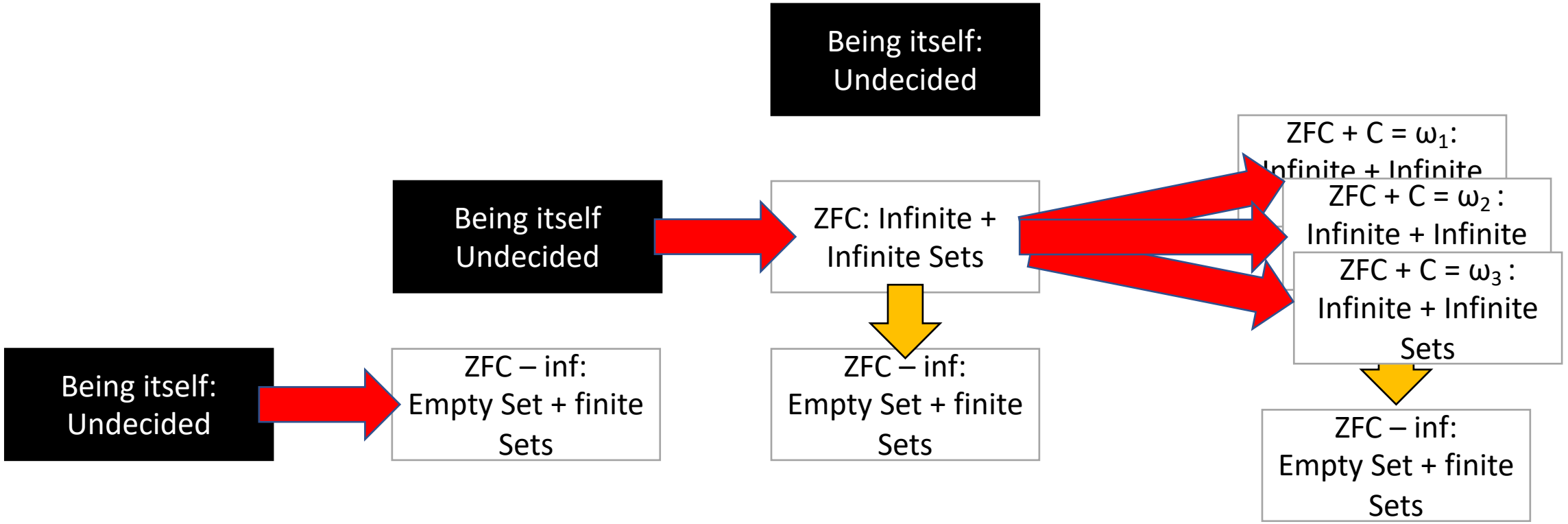
3.3.2 Constructivism, Idealism and its Negation

- V=L is a Axiom which states: There are only *constructible* Sets.
- This Axiom is discussed by Gödel to proof the *consistency* of ZFC with the continuum hypothesis.
- For Badiou, this Axiom has a specific philosophical significance. Its the *constructivist orientation*. It is associated with
  - 1. Idealism, Nominalism, Skepticism
  - 2. Reduction to language and knowledge
  - 3. Ideallingustics (in *Theory of the subject*).
- On the other hand V≠L states, there could be *generic sets, the « generic orientation »*. *T*he *Negation* of construcitivism can be associated with:
  - 1. Materialism, Platonism,
  - 2. Possibility of thinking the things in themselves and proper truths
  - 3. Materialism (in *Theory of the subject*)
- We see – turning back to the beginning - that the question of V=L or not is associated here with the old, the oldest philosophical question of idealism and materialism, the big question of Lenin and Althusser. This is a *translation* of the philosophical question into a mathematical question.
- Generally, the axiom V=L is considered by platonist mathematicians to be *restrictive* and by the »hardcore realists » *therefore* to be « wrong ». But there are also new « problems » of V≠L that show up.

3.3.3 Forcing, Continuum and Constructivism

- What is the main argument? *Only* if V≠L holds, Forcing is possible, and generic sets are possible.
- *Forcing* allows us to *demonstrate* that there are additions to our set theory, to show that a sentence is in the sense of Gödels *first incompleness theorem independent*. They are setteled by *generic sets*, whose existence can not be demonstrated or refuted in ZFC.
- So our universe is *larger,* if we allow forcing and generic sets and therefore take V≠L.
- In Being and Event the main focus is the forcing-method for demonstrating that the *size of the continuum* is independent from ZFC. We can *add* the size of the continuum as an additional axiom to ZFC to be more determinate on that question.
- But there are also other interesting applications. For example the *axiom of choice* itself is shown to be independent by forcing.
- Generally, forcing opens up the possibility of *more decisions.* The universe with V≠L is not only larger, but also *less determinate*.
- This conception is *not* heideggerian, because it enables the *intelligiblity* of that what can be added as an axiom *in* the « old world ». In that sense its more *platonic,* but a platonism of *decisions (Axioms).*

3.3 Add Axioms found by Forcing



3.3.4 « Decision to have decisions »

- Main Argument: We can think « more » if we allow V≠L and Forcing, therefore, V=L must be wrong. So it seems: the problem of « Heideggers uncertainty » of arbitrary »Weltanschauungen» is over.
- But: With forcing can see *more* possible axioms and decisions. And in this example its clear: Putting a specific cardinality of the Continuum with a new axiom is *not at all* a intuitive axiom! Here Gödels position, that set theory is build upon intuition, is coming to an clear end, what explains Badious emphasis on forcing.
- *Forcing* opens up a new *plurality* of possible mathematical axiom systems that *define further* our set theory, *which can not be decided on any reasonable intutive basis*. They are « beyond » our *intuitive* grasp, but *inside* our thinking and our ability to *decide* on them.
- Note: Even its proven the size of the continuum is independent of ZFC, it may be setteled in the future in the sense, that we can find important extensions to ZFC which decide it!

3.4.1 The Danger of the mathematical Multiverse

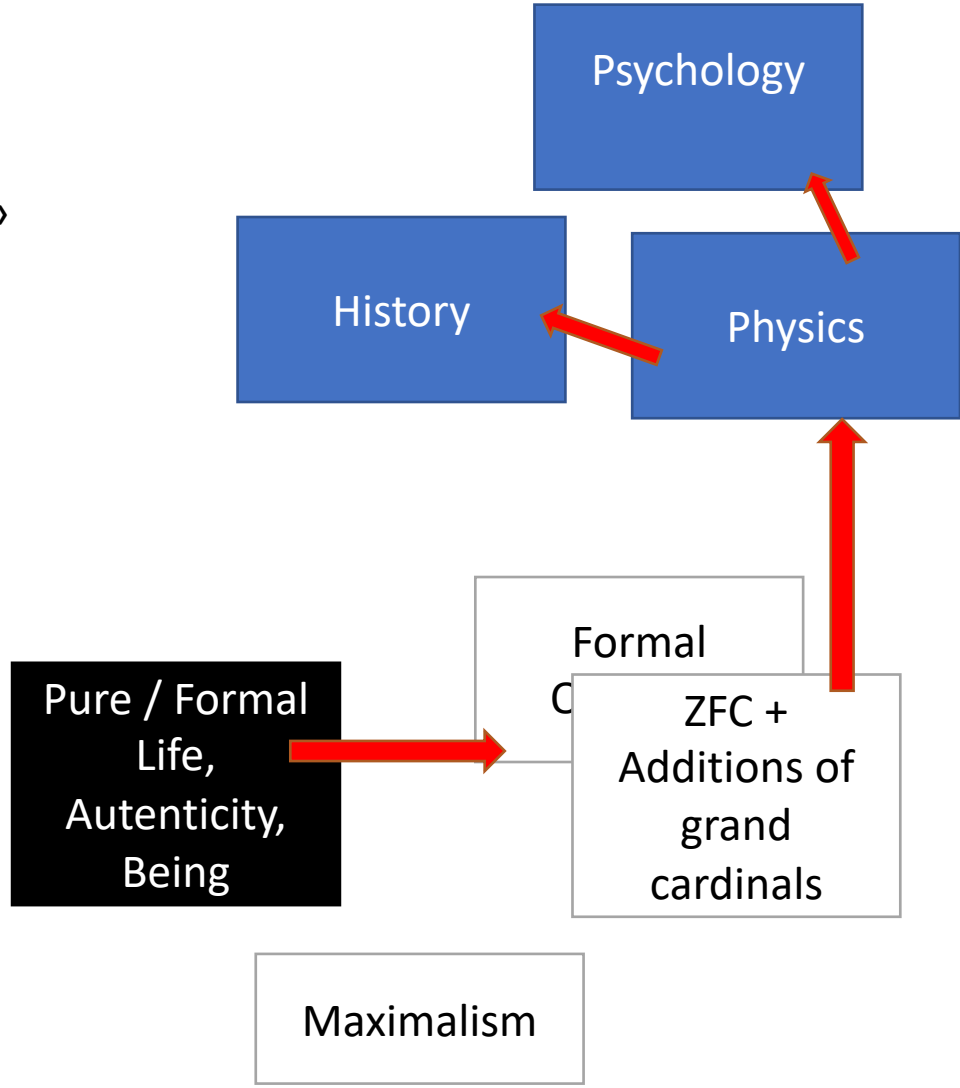
- Its clear, that forcing opens up *alternative* set theories as extension of ZFC, set theories, in which the continuum has a *specific* size. The axioms seem arbitary. What is the « true » set theory?
- Also more generally speaking, the *danger* of axiomatized mathematics is that somehow *every consistent system seems to be equally valuable*.
- Regarding set theory, this is called the *mathematical multiverse,* and says: There is no « true » set theory or its impossible to decide whats the « true » set theory. This is clearly what we can recoginize as *analogous* to the relativism of Dilthey and Heidegger of the *Weltanschauung*. So Heidegger comes back here, through the backdoor.
- The danger is: we search just the axioms that we need for our purposes!
- Badiou is explicitly against the idea, that there should be « many » mathematics, it has to be *one* (a typical platonist position) But how? Badious stance on these questions can be used to get out of this « Multiverse ».

3.4.2 Solution in *Transitory Ontology*, *Immanence of Truths*

- One idea against the danger *multiverse*, that Badiou affirms in a remark of *transitory Ontology,* in *Immanence of truths* is *« ontological maximalism »* or the *principle of maximality;* also his *ethics*.
- This principle / ethics can be interpreted as the idea, that the ***set theory*** that has so be searched, that can *describe as much sets as possible*. Its not an axiom, but a kind of *meta-axiom.* Badiou says *explitly*, that it guided already also *Being and Event 1*.
- With this meta-axiom two important things can be accomplished:
  - 1. We can deduce important *existence* axioms, also those that are *not* intuitive: The axiom of infinity, but also *all* grand cardinal axioms, this happens in *Immanence of truths*.
  - 2. We can decide that V=L (and idealism) has to be wrong, because then only the constructible sets exist. Because of this, also in *Being and Event* this concition seems to be *implicit*.
  - 3. We can possibly deduce even the « basic axioms » of set theory.
- One big advantage: The *ontological maximalism* is also a metaprinciple that doen't need any *intuition,* its only *symbolic*. We just want *to think as much sets as possible*.
- Also Penelope Maddy in some of her works sets the metaprinciple: *Maximize!* Also other philosophers of Mathematics defend ontological maximalism

3.4.3 Maximalism as Dissolution of Heideggers Problem

- Even if also other Philosophers stated this principle, and it was implicitly used by Mathematicians, with Badiou it's possible to read it as a *solution* Relativism / Irrationalism that Heidegger posed. With it its possible to determine the « true world » or the « true foundation » - its the world in which most things can be described scientifically, without becoming *inconsistent*. *All* infinities have to exist, and the finite world is « in » the infinite world.
- This helps not only to understand better mathematics and the problem of Heidegger, but also generally the idea of *foundation of science* and the fight against irrationalism and relativism.
- But There are also a lot of problems with it.
  - 1. It can not always be appplied that easily as with the large cardinals; it may come the moment who it will *not* be applicable at all,
  - 2. it is itself an arbitary axiom and therefore and the multiverse thesis can not stricly be « refuted ».
  - 3. Ultimately the metaprinciple-question is tied to the question what we want to describe in set theory / math:
    - 1. A Foundation Theory where everything is? 2. The world of intuitive sets? 3. A determined world of sets?
- Heidegger and relativism can never be *properly* refuted. But you can show, that good alternatives exist that are *not* arbitary.



3.5.1 Open Questions: AC and AF and Maximality

What remains in the « defence » of ZCF are two important axioms, which dont seem at all « intuitive », have a high importance and are also the « exit » of intuitionism / intuitionistic set theory:

1. Axiom of Foundation
2. Axiom of Choice

A. They are *weaker consequences* of V=L, but not this strong that they block the possibility of Forcing. At the same time, they and their consequences are very useful in mathematics

1. For example the *Los theorem* enables Forcing. That is, without a weaker Form of AC Forcing would also be impossible.
2. Completeness of Gödel, Stone Duality and others
3. Intermediate Value theorem (which is crucial for analysis)
4. Law of the excluded third / middle (Tertium non datur).

B. They seem to make the world of sets more « determinate », and allow key mathematical operations for all fields. The motivation for using them is (seemingly) *different* from ontological maximalism. Badiou says nontheless, that the AC is necessary in regard of the maximality principle; but its more complex than that.

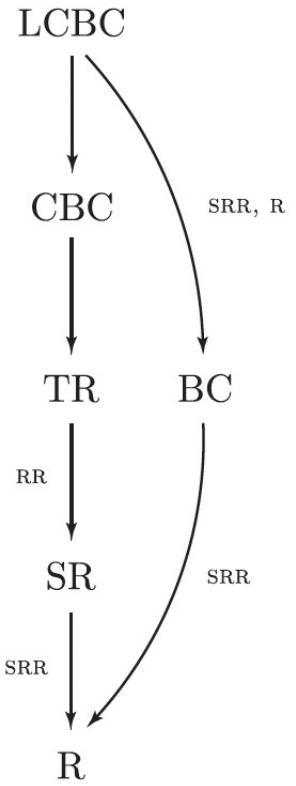
C. They can be proven to be *indepentent* from the other Axioms of ZFC, proven by cohens Forcing and permutation models

3.5.2 Kunen Theorem, ”Choiceless” Cardinals and AC

- One point of special importance is the *Kunen Theorem*, which is broadly discussed in *Immanence of Truths* as the »highest point « of V and the « ontological referent »
- Simply speaking it asserts, that a sufficient large Cardinal called the *Reinhard Cardinal* is inconsistent with the Axiom of Choice. Also more even larger Cardinals – Berkeley Cardinals – are inconsistent with AC.
  - 1. Usually, this is considered as a kind of « upper limit » of large cardinals, because AC is essential. Badiou does this also.
  - 2. *But* its also possible to consider another option: Drop AC and take these « super large cardinals » as existing! Then AC would be like V=L an « inner model », and a limit that has to be surpassed.
  - 3. In fact, ontological maximalism brings us « beyond » AC!
- This is in fact an conflict between « ontological maximalism » and « determinism » as maxims for the set theory; and if we take the usual way and that of Badiou, *eventually* they take determinism as more important than ontological maximality!
- So the truth is, that the « upper limit » at the Kunan theorem is *in between* of two tencencies, which lead to two different « directions »
  - 1. One direction of *maximal determinacy* leads to V=L (or Ultimate L), in which only a few large cardinal axioms can be valid.
  - 2. The « in between way » of setting the limit at the validity of AC and the Kunan theorem als Badiou does
  - 3. The *ontological maximalism* which leads to all « Berkeley Cardinals » which contradict also AC.

3.5.3 « Choiceless » Hierarchy beyond the Kunen Theorem

- But why this limit? Why is AC this important? Because we just want to use a *specific* mathematics with AC and specific cardinals? ---
- In the *balancing* of both drives – *Maximality* and *Determination* seems to be the only way to avoid Relativism and therefore the multiverse Problem. The *utility* of AC for specific mathematical questions and human purposes which Badiou mentions *can not* be an argument.
- There are also important open questions here in set theory, regarding the HOD-Conjecture, Consistencies of Berkeley Cardinals, the measurement of their consistency strength etc. Its also possible that »most» or « enough » of AC can be regained for everything that is really necessary in mathematics.
- There is also the possibility that AC has a deep philosophical significance, like avoiding Correlationism!
- So the question of the « ultimate set theory » V remains open and / or needs more research and philosophical principles.



From: Large Cardinals Beyond Choice
Joan Bagaria, Peter Koellner, and W. Hugh Woodin

4.1 Main Points:

- 1. If Heidegger concieves contemporary irrationalism in the way that science relies on *decisions*, but there are no *reasons* for it, in Marxism (specially in Lenin and Althusser) there was always the tendency to *acknowlege* this necessity of a decision and to *decide* these decisions towards *more science*.
- 2. If Heidegger is unable to see in science something that acts with decisions, that properly *thinks*, *Badiou* sees in *axiomatic set theory* something that works with decisions (axioms), and properly *thinks* in the sense of Heidegger (in fact it works like Heidegger concieves *metaphysics*).
- 3. If Heidegger is not able to give reasons why to take one decision over another – the disorientation itself -, Badiou finds in the development of set theory the principles of *ontologic maximality* and *determination* basic as « metaaxioms » in which direction the development has to be made. This is in harmony with the *marxist* solution for this problem.
- 3. If Heidegger can not decide on Materialism / Idealism, Badiou can decide against V=L and towards V≠L with this metaprinciple of maximalisation.
- 4. This « metaaxioms » dont work always. In the size of the continuum questions remain undecided. But these « open points of decision » are sometimes *intelligible* by methods like *forcing*. Also the metaaxioms can come to mutual contradictions like with the « choiceless hierarchy. »
- 6. With this principles a foundation of mathematics and science *can* be done and *is* being done.
- 7. With this principles its possible to make a *counterargument* to open and secret Heideggerianism (Latour, Morton, Malabou, Dugin and many more).

4.2 Three main positions

« Classical » Philosophy and Gödels Thinking on Mathematics	Heidegger / Dilthey / Multiverse Theory	Marxists / Badiou / Maddy
A. Metaphysics / Mathematics can accomplish self-foundation (Hilbert) B. Axioms of science and ontology are guided by *intuition* 1. Noetic intuition (Plato, Gödel) 2. Empiricial intuition 3. « Pure » sensible inutition (Kant)	A. No ultimate self-foundation B. Intuition is not sufficient for the axioms of our theories, they are *decisions*.	A. No ultimate self-foundation B. Intuition is not sufficient for the axioms of our theories, they are *decisions*. *C.* *Metaprinciples* like maximalism and determinacy guide the search for axioms. They are « decisions to decide less arbitarily »
There is *one true* science, which is built upon the « true » set of axioms that can be concieved by direct or indirect intuition. **= Impossible to hold**	There are different sciences and a multiverse of sciences which have « inner » ideas of truth and falsehood. The easy way out	Return to « one » set theory, one mathematics, and one science Has a lot of difficulties, is somehow « conservative »

Thanks